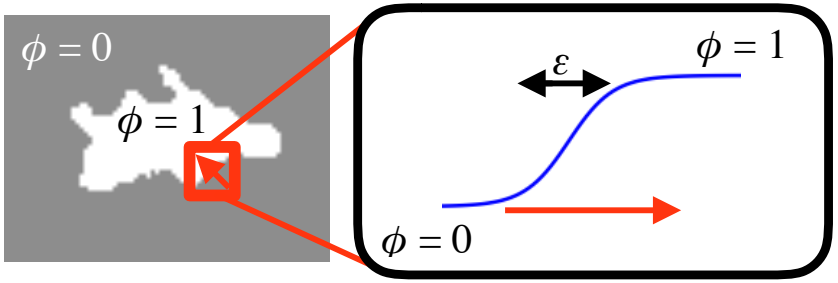
Phase-field model based on the reactions with excitability and polarity



Simulations

Reaction in a cell (Reaction Diffusions) V, U, W ⇔ Membrane deformation ϕ

$$\tau\frac{\partial\phi}{\partial t} = \eta\left(\Delta\phi - \frac{1}{\epsilon^2}G'(\phi)\right) - M\left(\int \phi d\boldsymbol{r} - A_0\right)|\nabla\phi| + F_{\mathrm{prot}}|\nabla\phi|$$

$$F_{\mathrm{prot}}=a_{\mathrm{w}}W$$

$$\frac{\partial(\phi V)}{\partial t} = \phi\left(\alpha U\frac{WV}{K_{\mathrm{k}}+\frac{\int d\boldsymbol{r}^2\phi WV}{\int d\boldsymbol{r}^2\phi}} - \beta V\frac{U}{K_{\mathrm{p}}+\frac{\int d\boldsymbol{r}^2\phi U}{\int d\boldsymbol{r}^2\phi}} - \mu V\right) + \phi N(\boldsymbol{r}) + D_{\mathrm{V}}\nabla\cdot(\phi\nabla V)$$

Noise-term

$$\frac{\partial(\phi U)}{\partial t} = \phi\left(-\alpha U\frac{WV}{K_{\mathrm{k}}+\frac{\int d\boldsymbol{r}^2\phi WV}{\int d\boldsymbol{r}^2\phi}} + \beta V\frac{U}{K_{\mathrm{p}}+\frac{\int d\boldsymbol{r}^2\phi U}{\int d\boldsymbol{r}^2\phi}} + s - \gamma U\right) - \phi N(\boldsymbol{r}) + D_{\mathrm{U}}\nabla\cdot(\phi\nabla U) - \chi_{\mathrm{u}}\frac{U|\nabla\phi|^2}{\int d\boldsymbol{r}|\nabla\phi|^2}$$

Noise-term

Absorption rate of U on the edge

$$\frac{\partial(\phi W)}{\partial t} = \phi\left(k_{\mathrm{W1}}(-\rho W^3 + \rho W_{\mathrm{cyt}}W^2 - W) + \zeta V\right) + D_{\mathrm{W}}\nabla\cdot(\phi\nabla W)$$

The meaning of the Parameters

(Characters in the code) → (Parameter in the equations)

Du → D_U, Dv → D_V, alpha → α, Kk → K_K, beta → β, Kp → K_P, s → s,
gamma → γ, mu → μ, kaiU → χ_U, kaiV → χ_V
theta → θ, d → d, sigma → σ
k_vz → ζ
Dz → D_Z, gw3 → $k_{W1}\times\rho$, kw2 → $k_{W1}\times\rho$, gw1 → k_{W1}, Ntot → W_{tot}
tau → τ, eta → η, epsilon → ε, M → M, aw → a_W, A_0 → A_0